ISSN 1870-4069

Vector Theory for the Scattering of TM-polarized Hermite-Gaussian Electromagnetic Beams by a Double Metallic Slit

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Abstract. We present a rigorous theory for oblique incident Hermite-Gaussian beams, diffracted by two slits of width ℓ and separation d, in a thick metallic screen for the case of polarization TM(S). The far field spectra as a function of several opto-geometrical parameters, wavelength λ, slit width ℓ, separation d, incidence angle θ_i and Hermite order m is analyzed. In the vectorial diffraction region given when $\lambda/\ell > 0.2$, where ℓ is the incident wavelength and as a function of the separation between slits d; we have numerically analyzed: the far field spectra, the energy diffracted along the incident beam direction (E_i), and the validity of an approximate diffraction (scalar) property, namely $E_i = N\tau/\lambda$.

Keywords: diffraction, scattering, double slit.

1 Introduction

Currently there are several rigorous theories of diffraction by plane electromagnetic waves (Enriquez *et al.*, 2011) and Gaussian beams (Mata *et al*, 1993); (Mata *et al*, 1994) by two slits in metallic screens of zero thickness. However these theories do not treat with Hermite-Gauss or oblique incidence, nor thick screens of nonzero thickness (Mata *et al*, 2008).

In this paper we present a novel rigorous theory of diffraction that allows to consider the illumination by Hermite-Gaussian beams at oblique incidence on two slits of width ℓ and separation d in screens with infinite conductivity and thickness h.

In particular, we analyze the coupling between slits through the numerical study of the diffracted energy along the direction of the incident (E_i) beam energy as a function of the parameter of separation d between the slits. It is revealed the existence of oscillations in the energy E_i . We also show that in the case of TM(S) polarization, the energy E_i is special because when compared to other diffraction patterns. Finally, we show that the scalar property valid at the scalar region (λ/ℓ <0.2) $E_i = N\tau/\lambda$ (Alvarez-Cabanillas, 1995) is not longer valid.

2 A Vector Theory of Diffraction

In Fig.1 we have two slits on a screen of infinite conductivity, and non-zero thickness denoted by h. In this screen you have two parallel to the Oz axis, ℓ wide and spaced slits d. The display is in the gap and impinges perpendicularly on it a Hermite-Gaussian beam with wavelength $\lambda = 2\pi/k$ and order m. We will use the complex representation for the fields and omit the time factor going forward $e^{-i\omega t}$. H is the magnetic field when you have the TM (magnetic field parallel to the axis Oz) polarization. The H field satisfies the Helmholtz equation (Mata *et al*, 1994)

$$\partial^2 \mathrm{H}/\partial \mathrm{x}^2 + \partial^2 \mathrm{H}/\partial \mathrm{y}^2 + \mathrm{k}^2 \mathrm{H} = 0. \tag{1}$$

Denote by H_I the solution of Eq (1) in the region I ($y > h/2$), expressed by a plane wave expansion:

$$\mathrm{H_I(x,y)} = \frac{1}{\sqrt{2\pi}} \int_{-k}^{k} \mathrm{A}(\alpha)\, \mathrm{e}^{\mathrm{i}(\alpha x - \beta y)} \mathrm{d}\alpha + \frac{1}{\sqrt{2\pi}} \int_{-k}^{k} \mathrm{B}(\alpha)\, \mathrm{e}^{\mathrm{i}(\alpha x + \beta y)} \mathrm{d}\alpha. \tag{2}$$

The first integral is identified with the incident beam due to the sign of the α and β k-components.

In region II, within the screen, $-h/2 < y < h/2$ the electromagnetic field will be represented by the following modal series:

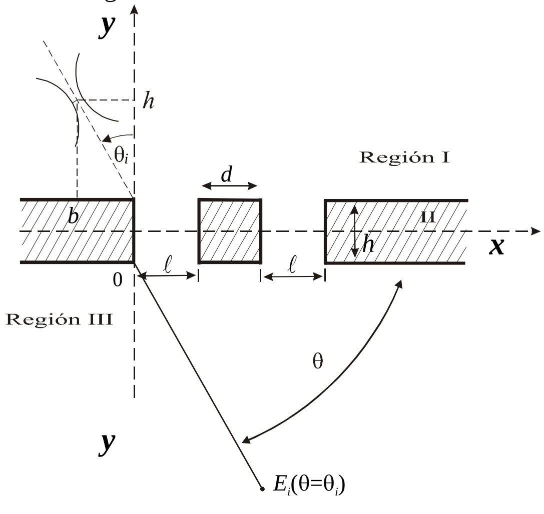

Fig. 1. Our system. Two slits of width ℓ and spacing d in an infinitely thick conducting screen h. The energy diffracted along the incident direction (E_i) is diffracted in the direction of θ(relative to the axis Oy) = θ_i (From the axis Ox).

$$\mathrm{H_{II}(x,y)} = \sum_{\mathrm{n=0}}^{\infty} \mathrm{a_n^1 \varphi_n^1(x,y)} + \sum_{\mathrm{n=0}}^{\infty} \mathrm{a_n^2\, \varphi_n^2(x,y)}, \tag{3}$$

where in $i = 1{,}2$ the set $\varphi_n^i(x)$, are functions whose normal derivative is zero at the walls for the TM polarization.

The diffracted field below the screen, for $y < -h\,/\,2$ (region III), could be expressed as a plane wave expansion too:

$$H_{III}(x,y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} C(\alpha)\, e^{i(\alpha x + \beta y)} d\alpha. \tag{4}$$

Our goal is to determine the transmitted field (Eq. (4)), for which one needs to determine $C\,(\alpha)$. Note that $C\,(\alpha)$ depends on the coefficients a_n^1 and a_n^2 and the incident amplitude $A(\alpha)$. For this, we use the appropriate conditions of continuity, which could be obtained from Maxwell's equations (Alvarez-Cabanillas, 1995). These conditions lead us to the following matrix system, in which the matrix columns $\boldsymbol{a_1}$ and $\boldsymbol{a_2}$ are formed respectively by the coefficient a_n^1 and a_n^2.

$$\begin{aligned} M_{11}a_1 + M_{12}a_2 &= S_1, \\ M_{21}a_1 + M_{22}a_2 &= S_2, \end{aligned} \tag{5}$$

where M_{ik} $(i,k = 1{,}2)$ are square matrices dependent on the opto-geometrical parameters and S_i $(i = 1{,}2)$ are matrices depending only on $A\,(\alpha)$. The determination of the modal coefficients a_n^1 and a_n^2. allow us to calculate the diffracted field in any region for TM polarization.

3 Results and Discussion

Using the complex Poynting vector is possible to obtain the diffracted intensity at the angle θ. For a Hermite-Gaussian beam, the spectral amplitude is (Mata *et al*, 2008):

$$\mathrm{A}(\alpha) = \frac{\mathrm{L}}{2}\mathrm{i^m H_m}\left[-\frac{\mathrm{L}}{2}(\alpha \sin\theta_\mathrm{i} - \beta\cos\theta_\mathrm{i})\right] \times \left[\sin\theta_\mathrm{i} + \left(\frac{\alpha}{\beta}\right)\cos\theta_\mathrm{i}\right] \mathrm{e}^{(-\mathrm{i}\alpha \mathrm{b})} \times \mathrm{e}^{[-(\alpha\sin\theta_\mathrm{i} - \beta\cos\theta_\mathrm{i})^2 \mathrm{L}^2/8]}, \tag{6}$$

where H_m is the Hermite polynomial of order m. The position of the beam waist is given by the parameter b (see Fig. 1).

In the figures relating to energy diffracted along the direction of the incident beam is $E_i(\theta = \theta_i)$ the diffracted angle in the direction of the incident beam, measured from the axis Ox and θ_i is the angle of incident beams to the axis Oy measured. The energy, the diffracted intensity $I(\theta)$ and the transmission coefficient τ are normalized to the total incident energy I_0. All parameters normalized opto-geometrical width lof the slots ℓ, that is, , ℓ= 1.

In Figs. 2 and 3 show the diffraction patterns of Hermite-Gaussian beams for the fundamental mode $m = 0$ at normal incidence and oblique incidence of $30\,°$; the wavelength of the incident beams is λ/ℓ=0.9, with extremely wide Gaussian beams L/$\ell = 500/\sqrt{2}$ and fixed at the position b/ℓ=0.5, the thickness of the screen is h/ℓ =1and the gaps between slits are d/ℓ= 0, 1, 3.5 and 5.

The shape of the diffraction patterns for the $m = 2$ mode, not shown, is identical to the spectra of FIGS. 2 and 3 (with the same opto-geometrical parameters) except for a scaling factor which provides a lower intensity for this mode, from the respective Hermite polynomial.

From these diffraction patterns we have taken the diffracted energy E_i along the direction of the incident beams. Figs. 4 and 5 show the behavior of the E_i separation according to d for $m = 0$ and 2 modes; opto-geometrical parameters of these figures are the same in Fig. 2 and 3.

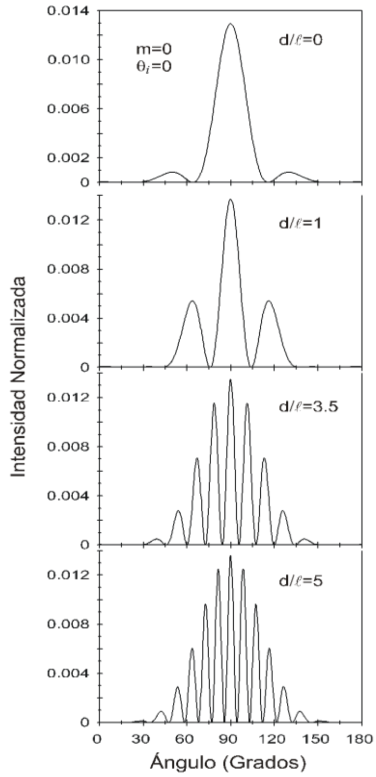

Fig. 2. Diffraction patterns normalized $(I(\theta)/I_0)$ of Hermite-Gaussian beams of $m = 0$ normally incident on two slits so. With λ/ℓ = 0.9, L/ℓ=500/$\sqrt{2}$, h/ℓ =1 and position b/ℓ=0.5 and for separations d/ℓ = 0, 1, 3.5 and 5.

The curves of FIGS. 4 and 5 show the oscillatory behaviors as E_i a function of the spacing d, in particular for the period is normal incidence to oblique incidence λ and the period is 2λ.

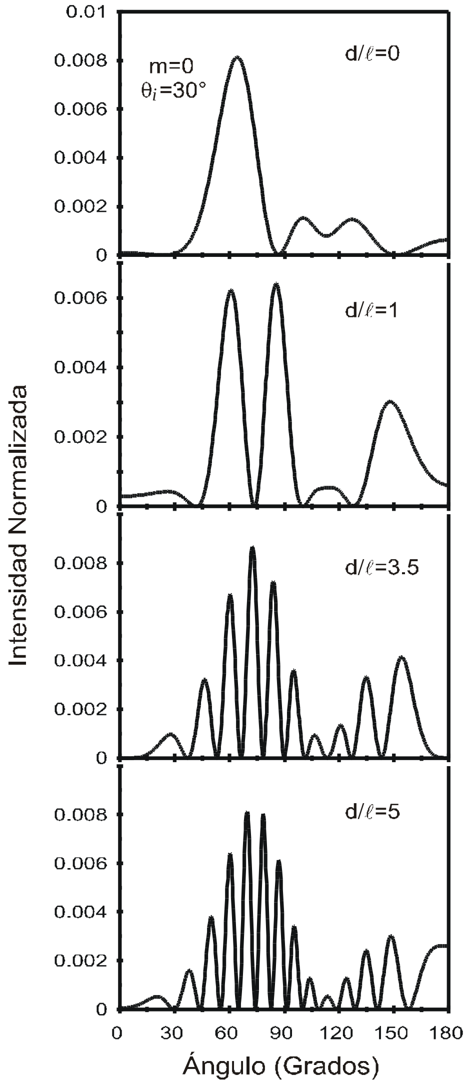

Fig. 3. Standard diffraction patterns $(I(\theta)/I_0)$ Hermite-Gaussian beam for $m = 0$ to $30\,°$ obliquely incident on two slits so. Same parameters of Fig.2.

In Fig. 5 has also been drawn in broken lines the 2τ/λ function. As you can see, this function does not overlap with the energy with E_i which we can say that the property of diffraction E_i =2τ/λ is not valid in the vector region at least for the separation parameter d and doing extremely wide.

Finally, in Fig. 6 different diffracted energy around the energy is E_i. The upper curves of Figure 6 correspond to normal incidence for the $m = 2$ mode, with the same parameters of Fig. 3; diffracted energies correspond to the angles diffracted $\theta = 90°, 91°, 92°\ and\ 94°$. The curves in the lower window of Fig. 6 correspond to oblique incidence of 30°, also for mode $m = 2$, with the same parameters of Fig. 4. The diffracted energies shown, corresponding to angles diffracted around of $\theta = 60$ ° (corresponding to the diffracted energy along the oblique incidence angle $\theta_i = 30$ °) and for the angles 58 °, 57 ° and 64 °.

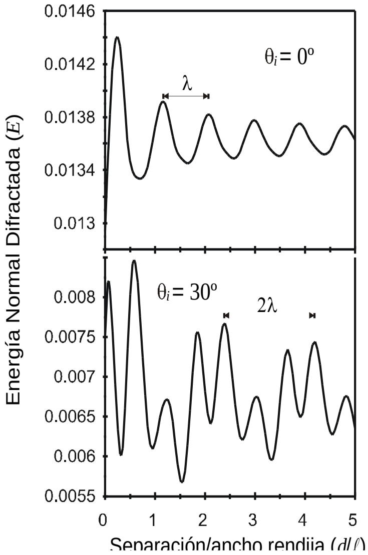

Fig. 4. Energy diffracted in the direction normal to the standard E_i to Hermite-Gauss beam, depending on the spacing d/ℓ screen. For the fundamental mode $m = 0$, at normal incidence and oblique incidence of 30°, with λ/ℓ=0.9, L/ℓ=500/$\sqrt{2}$, h/ℓ =1, y b/ℓ=0.5.

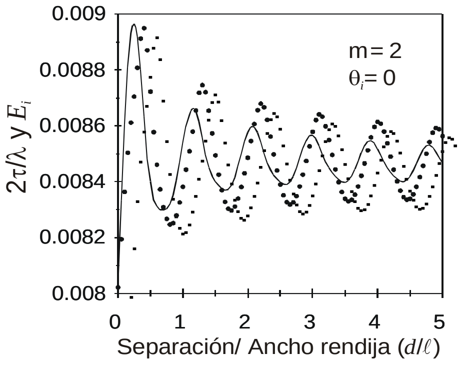

Fig. 5. Energy diffracted in the direction normal to the E_i (solid line) Hermite-Gauss beam, thus $m = 2$ and $2\tau/\lambda$ property (dashed line), in function of the spacing d/ℓ. Same parameters of Fig. 3.

Energy analyzing energy diffracted E_i around for $m = 0$ at normal incidence and oblique incidence of 30 ° as also carried out (data not shown) found similar patterns for mode $m = 2$ (see Fig. 6), the energy diffracted around the energy as E_i a function of the spacing d, decay to zero.

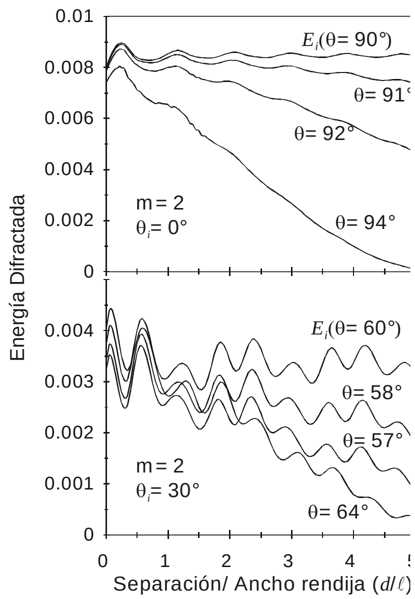

Fig. 6. Energy diffracted around energy $E_i(\theta = \theta i)$ Hermite-Gauss beam, for the $m = 2$ mode according to the distance between slits d/ℓ. Same parameters of Fig. 4.

4 Conclusions

Present a more rigorous theory of diffraction for the oblique incidence beam Hermite-Gaussian (HG) on a screen of thickness h with wide slits separating slits ℓ and d. In the case of TM(S) polarization and wavelengths in the vector region $\frac{\lambda}{\ell} > 0.2$, we have found that the diffracted along the direction of the incident beam energy has oscillations period λ as a function of the spacing d for modes $m = 0$ and 2, for the period 2λ at $30\,°$ oblique incidence. Finally, we note that the energy E_i has special characteristics compared diffracted energies in other directions and found numerically that ownership of scalar diffraction $(\lambda/\ell < 0.2)$ given by $E_i = 2\tau/\lambda$ is no longer valid in this region $(\lambda/\ell > 0.2)$.

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